

# g-FACTOR AND GRADIENT WEIGHTED DENOISING WITH EDGE RESTORATION (g-DENOISER) FOR SENSE RECONSTRUCTED MR IMAGES

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## ABSTRACT

The application of partially parallel imaging techniques to regular clinical MRI studies has brought about the benefit of significantly faster acquisitions but at the cost of amplified and non-uniformly distributed noise, especially, for high acceleration factors. In this work, denoising of the images reconstructed by the sensitivity-encoding (SENSE) algorithm is presented. To efficiently remove noise and simultaneously preserve the image details, weighted total variation smoothing with weighted edge restoration has been developed. Denoising is guided by the g-factor of the SENSE reconstruction and image gradient that is representative of edge information. An automatic iteration termination scheme is proposed to balance denoising and edge preservation, and reduce the difficulty of parameter decision in conventional approaches. The proposed g-factor and gradient weighted denoising with edge restoration (g-DENOISER) method retains the image details better than the denoising techniques guided by only the g-factor and simultaneously suppresses the noise stronger than the techniques where g-factor was not used to adaptively adjust the denoiser parameters according to local noise characteristics. It can be effectively used to reduce noise in images acquired using high acceleration factors and reconstructed using SENSE.

**Keywords**— SENSE, g-factor, gradient, denoising and edge restoration

## 1. INTRODUCTION

Partially parallel imaging (PPI) techniques [1-3] are being routinely used to achieve increased image resolution, decreased motion artifacts and to shorten scan time. However, PPI techniques reduce acquisition time at the cost of loss in signal to noise ratio (SNR). With increase in acceleration factor, the increase in noise can be significant thereby reducing the diagnostic quality of the image. Most smoothing techniques employ a low pass filter to preserve the signal strength in the low frequency regions, since noise is prevalent in the high frequency regions. However, edges are also stored in the high frequency region; low pass filters for denoising cannot distinguish between edges and noise. To protect edge information, anisotropic smoothing techniques such as the total variation model [4] and spatially varying diffusion filtering [5], were introduced. Though better than conventional filtering techniques, the anisotropic techniques can still damage the edges while trying to clean some excessively noisy regions or suffer from excessive unfiltered noise in low-SNR regions.

There are two approaches to solve this problem. The first is to design filters that use different weights for smoothing different regions, i.e. noisier regions and regions without edges are smoothed more to remove noise better; less noisy regions and regions with edges are smoothed less to protect the edges better.

Several techniques based on this approach have been developed [6-8]. The second approach is to restore edge information that may be lost during the smoothing process. The residual, which is the difference of the original image and the resulting smoothed image, often has both noise and edge information. However, incorporating the residual back into the smoothing process can restore the edge information [9]. In this technique, after every stage of smoothing, the residual is added back to the original image and the new edge enhanced initial image is smoothed again.

In this work, a g-factor and gradient-guided total variation filtering technique [10] to denoise the images and add back the lost information (which includes both noise and edge information) weighted by the g-factor (which is representative of the noise levels in the image) and the gradient (which is representative of the edge information) is introduced. An automatic scheme is proposed to terminate the iteration. This helps to strike a balance between smoothing and edge preservation, thereby leveling the noise distribution while still protecting edges. The proposed method is named the g-factor and gradient weighted denoising with edge restoration (g-DENOISER) technique. Results from experiments performed on phantom data and brain data are presented.

## 2. THEORY

### 2.1 Existing Techniques:

The conventional TV model [7] is described by:  $\min E[u|u_0]$  where

$$E[u|u_0] = \int_{\Omega} \lambda(u - u_0)^2 dx + \int_{\Omega} |\nabla u| dx \quad (1)$$

If the denoising problem is considered as an input-output system, then the input is a noisy image,  $u_0$ , and the output is the denoised image,  $u$  which minimizes the energy functional  $E[u|u_0]$ .  $\Omega$  is the image domain. The g-factor 'g' is information obtained from the SENSE reconstruction and can be used as a criterion for the minimization of the expression in the fidelity term – which is the first term in equation 1. To control the amount of smoothing in any given location in the image, the balance between the smoothing and fidelity terms is made locally adaptive to both the spatially varying noise level and the likelihood of an edge by setting  $\lambda$  to be a function of both the image g-factor and gradient. Regions where the g-factor is higher and the gradient is weaker get smoothed more and regions where the g-factor is lower and the gradient is stronger get smoothed less. To control the type of smoothing, in [8] the authors proposed a modified TV model where the exponent,  $p$ , in

the regularization term  $\int |\nabla u|^p$  varies spatially so the smoothing is isotropic or anisotropic depending on the strength of the gradient. Therefore, both the amount and type of smoothing can be made spatially adaptive [8] by using the minimization energy:

$$E[u | u_0, \nabla u_0] = \int_{\Omega} \lambda(x)(u - u_0)^2 dx + \int_{\Omega} |\nabla u|^{p(x)} dx \quad (2)$$

where  $p(x) = p(|\nabla u_0(x)|)$  is a decreasing function of the magnitude of the gradient of the initial data  $u_0$  which takes on values between 1 and 2 such that  $\lim_{|\nabla u_0(x)| \rightarrow \infty} p(x) = 1$  and  $\lim_{|\nabla u_0(x)| \rightarrow 0} p(x) = 2$ .

Therefore, the filtering is isotropic in ‘flat’ regions, and anisotropic elsewhere. Furthermore, the type of anisotropy varies depending on the strength of the gradient. At regions with very high gradient, i.e., likely edges, smoothing is strictly TV based and is only performed in the direction strictly tangential to edges. At more ambiguous regions with lower gradient, which could represent either weak edges or noise, anisotropic smoothing that is a combination of TV and isotropic regularization is used. In the g-DENOISER technique, we make use of Eq. (2) to implement the adaptive TV filtering, but the spatially varying parameters  $\lambda(x)$  and  $p(x)$  will now depend on the g-factor and the gradient information.

One side effect of models such as (1) is that there is a loss of contrast, and thus a loss of edge information that occurs from the denoising process. In [10], Osher et al. showed that by taking the solution,  $u$ , of the TV model (1), adding the residual,  $v = u_0 - u$ , back to the original data  $u_0$ , and using  $u_0 + v$  as new input for the TV model, one gets a new solution which is not only a denoised version of the original, but also has retained more edge information. This process can be iteratively repeated, resulting in images with increased noise and enhanced boundary, eventually an image similar to the original image. There is edge enhancement since, with each iteration, the input to the minimization has more edge information. But it is smoothed the same as it was initially, thereby emphasizing the edge information. This process of adding the edge information back is also used in the proposed g-DENOISER technique.

## 2.2 g-DENOISER

The g-DENOISER technique uses a combination of the modified total variation (TV) filtering as described in [8] and a modified version of edge restoration [10]. Both use the g-factor and the image gradient as guiding factors for the minimization and parameter estimation. The filtered image is obtained from the iterated minimization of the following expression:

$$u^{m+1} = \arg \min_{u^{m+1}} \left| \int \lambda(x) |u^{m+1} - u^{m*}|^2 dx + \int |\nabla u^{m+1}|^{p(x)} dx \right| \quad (3)$$

where the input to each iteration,  $u^{m*}$ , is defined as follows:

$$u^{m*} = u_0 + M \times (u^m - u^{m-1}) / (1 + M \times g) \quad (4)$$

which is equivalent to the original SENSE reconstructed image with a weighted version of the residual (edge and noise information) added back. In Eq. (4), ‘ $\times$ ’ denotes pixel-wise multiplication;  $g$  is the g-factor map as obtained from the SENSE reconstruction algorithm;  $u_0$  is the original SENSE reconstructed noisy image,  $M$  is the mask of boundary and defined by Eq. (5) below as explained in [5]:

$$M = 1 - \frac{1}{1 + \frac{|\nabla(G \otimes u_0)|^2}{\kappa}} \quad (5)$$

where  $\otimes$  is the convolution operator,  $G$  is the Gaussian convolution kernel,  $\kappa$  is the value of the bin that had the most number of elements in the histogram of the absolute value of the smoothed gradient. From Eq. (5), it can be seen that  $M$  is close to 1 at regions having high gradient, and tends to 0 at flat regions. From

Eq. (4), it can be seen that only the difference near the edge regions that have a low g-factor is added back. This definition makes sure that only the edge is restored. The noise in the flat regions can be efficiently removed. In Eq. (3),  $p(x)$  is defined as  $2 - M$ . This definition follows the requirement that  $p(x)$  tends to 1 at regions near edges, and tends to 2 at flat regions. This definition makes the *type* of smoothing adaptive;  $\lambda(x)$  is used to adaptively regulate the *amount* of smoothing, hence  $\lambda(x)$  is defined as:

$$\lambda(x) = \frac{\beta}{p(x)g(x)} \quad (6)$$

where  $\beta$  is a parameter set to an appropriate level depending on the image. According to Eq. (6),  $\lambda(x)$  is smaller where g-factor higher i.e. at regions needing more smoothing. With a smaller  $\lambda(x)$ , the smoothing term is emphasized more and hence a smoother result will be generated. The partial differential equation (PDE) defined by Eq. (3) can be solved by iteratively solving its Euler Lagrange expansion as derived in reference [8].

## 2.3 Flexibility of choice of parameters:

From Eq. (3) & Eq. (6), there is only one parameter,  $\beta$ . However, there are two other numbers of iteration that need to be defined. One is the number of iterations required to solve the PDE in Eq (3)(iteration 1), the other is for the iteration of edge restoration (iteration 2). A smaller  $\beta$  implies more smoothing in the overall image. In g-DENOISER, the choice of  $\beta$  is more flexible because over-smoothing is compensated. If  $\beta$  is too small, and the edge is damaged, the edge is added back during the iterations of edge restoration. The range of  $\beta$  chosen is 0.5 ~ 20. Similar to the explanation of the flexibility of the choice of  $\beta$ , the choice of number of iterations ‘ $N$ ’ for iteration 1 is also flexible. Hence ‘ $N$ ’ can be predefined. A larger ‘ $N$ ’ will take longer computation time hence ‘ $N$ ’ is fixed to be 30 in our implementation. Typically, values less than 100 work well.

## 2.4 Termination Criterion:

The number of iterations of iteration 2, which is for edge restoration, can be decided automatically. Since g-factor demonstrates the noise distribution in the image reconstructed by SENSE, it is understood that the removed noise should have a pattern similar to that of the g-factor. Hence the statistical similarity between the g-factor map and the removed noise is used for determination of the termination criterion. Before edge restoration, because the image is smoothed guided by both, the g-factor and gradient, the correlation between the g-factor map and the difference map (difference between the original noisy SENSE reconstructed image and the filtered image) is large at regions near the edges. With the increase in number of iterations of restoration, more edge information is added back and the correlation at the regions near edges reduces even more. The correlation values are compared with each iteration and the iterations are terminated when the correlation value increases to a value larger than the previous value.

## 2.5 Image Evaluation Criterion:

Image quality evaluation is not an easy task. The question of whether an image with more edge information and more noise is better than an image with reduced noise but damaged edge, or vice versa has no definite answer. To more accurately test the performance of the proposed method, two different methods from different points of view were used to evaluate the image quality. 1) *Relative error* defined as the ratio of the L2 norm computed from the difference images (difference between the outputs of each denoising technique and the original noiseless

image), to the L2 norm computed from the true original noiseless image. This value shows the similarity of the result and the gold standard. The smaller the number, the more similar (i.e. better in the sense of similarity) the result is to the original, true, noiseless image; 2) *Image sharpness* computed as defined by the authors in [12]. The local maximum and minimum intensity values across a boundary in the denoised were determined as  $I_{max}$  and  $I_{min}$ . The distance, 'd' between  $0.8(I_{max} - I_{min}) + I_{min}$  and  $0.2(I_{max} - I_{min}) + I_{min}$  across the profile was measured and the sharpness for that region evaluated as  $1/d$  ( $mm^{-1}$ ). This value demonstrates the edge protection. A larger number means a sharper image, (i.e. better in the sense of edge protection).

### 3. METHODS

Two different sets of experiments were performed to demonstrate the proposed method. Phantom images were simulated and processed with a set of parameters,  $\beta$  and N. Reduction factor 3 was simulated. A set of T2-weighted brain images were acquired on a 3T Trio scanner (Siemens Medical Solutions, Erlangen, Germany), with an 8-channel head coil (In vivo Diagnostic Imaging, Gainesville, Florida) and a dual contrast 2D turbo spin echo sequence (FOV: 220x165, matrix size: 320x245, slice thickness: 2mm; TR 4000ms, TE1/TE2: 12/99ms, ETL: 14, bandwidth: 130 Hz/pixel). Full k-space data was acquired and a SENSE acquisition/reconstruction with a reduction factor of 3 was simulated with this dataset to show the performance of the new technique. This brain dataset was corrupted with additional complex Gaussian noise added to it to show the effectiveness of the technique.

The sensitivity maps for SENSE reconstruction and g-factor estimation were obtained from the full k-space data, as the raw channel data normalized by the square root of sum of squares image. Edge detection for all experiments was performed using the Perona-Malik anisotropic diffusion method [5], with the initial low pass filtering being performed using a Gaussian filter.

The termination criterion was implemented by comparing the correlations regionally within the image. The difference image and the g-map were both divided into many regions of size close to 5x5 (depending on matrix size). Correlation was computed for every region between the mean of the g-map and the standard deviation of the difference image in that region. All regions that had a correlation of greater than 50 percent were counted for the criterion. With each iteration, the number of regions with a correlation greater than 50% was compared with that obtained from the previous iteration. This process was repeated till the number increased to a value more than the previous iteration. The edge restoration was then stopped.

### 4. RESULTS

The results for phantom and brain data are presented. A modified Shepp Logan phantom and an arbitrary 6-channel array (3 elements on top and 3 elements at the bottom) were simulated and used for SENSE reconstruction (see Fig 1). The first stage of low pass filtering for the edge detection was performed using a Gaussian filter. For the g-DENOISER technique, the parameters used were in the range:  $\beta = 0.5$  to 20 and  $N = 30$ . Different values of  $\beta$  were tried to illustrate the robustness of the algorithm to the choice of parameters. The edge information, also weighted by g-factor was added back and processed till the termination criterion was satisfied to give the final results. The same dataset was also

processed using g-factor guided denoising alone where identical values of  $\beta$  were tried and N varied from 30 to 90 to give the optimum result for this case. Also, the dataset was processed with gradient weighted filtering with edge restoration, where the g-factor map was set to unity during edge restoration. Therefore,  $\lambda$  was the same but the edge information being added back was not g-factor weighted. This was also done for  $\beta$  ranging from 0.5 to 20 and  $N = 30$ . All results shown are for  $\beta = 5$ . Figures 1 (d), (e) and (f) are the outputs of the images denoised using the new g-DENOISER, g-factor alone and gradient weighting alone respectively. The arrow indicates the region where there is better edge restoration in the new technique. The new combined technique clearly has a better denoising performance with edge restoration capability than either of the two techniques independently. Table 1 shows the relative error defined by the ratio of the L2 norm computed from the difference images obtained from the original SENSE reconstructed image, the new technique, g-factor guided denoising alone and gradient weighted filtering with edge restoration alone, to the L2 norm computed from the true original noiseless image. As seen, the g-DENOISER technique has the least value of the computed L2 norm ratio. The results indicate that the image denoised using the new technique definitely is the closest to the truth than the gradient weighted TV filtering with edge restoration alone or the g-factor guided TV filtering alone.

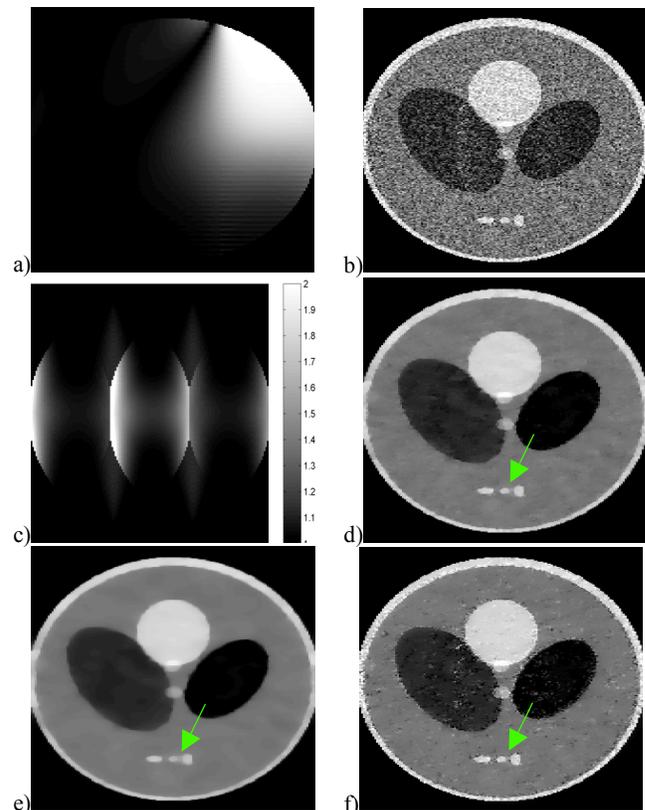


Figure 1. Phantom results (a) Sensitivity profile of one of the hypothetical coil elements (b) Original SENSE reconstructed noisy image (c) g-factor map generated for a reduction factor 3 (d) Output of the new g-DENOISER technique (e) Output of denoising using g-factor alone (f) Output of denoising using gradient alone.

Table 1. Relative error

	SENSE	g-DENOISER	g-factor guided filtering	gradient weighted filtering
Phantom	0.48	0.2051	0.2093	0.2598

Figure 2 illustrates the results of the denoising algorithm applied to the brain dataset. (a) shows the results of the g-DENOISER technique, (b) shows that of denoising using g-factor guided TV filtering alone, (c) shows the images denoised using gradient weighted edge restoration alone. (d) shows the zoomed in version of the region where the 2 arrows are. The arrows depict edge details where the images denoised using the proposed technique are visibly better than either of the two techniques used independently. The relative error computed for our proposed

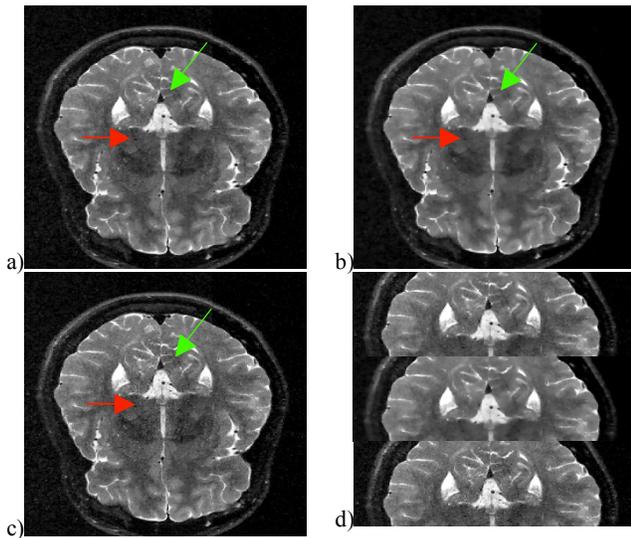
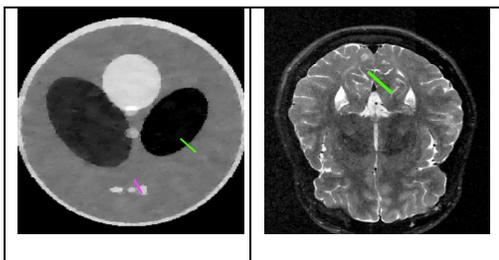


Figure 2. Brain data results (a) Output from the g-DENOISER technique (b) Output from denoising using g-factor alone (c) Output from denoising using gradient weighting alone (d) Zoomed in regions from all the above three images indicating the ROI (where the arrows are).



g-DENOISER	5.05 (m)	5.79 (g)	2.55
g-factor guided TV	4.42 (m)	5.67 (g)	1.74
gradient weighted TV	5.95 (m)	5.74 (g)	2.57

Figure 3. Image sharpness evaluation

technique, g-factor guided filtering alone and gradient weighted edge restoration alone are 3.37, 3.40 and 3.48 respectively. Figure

3 demonstrates the sharpness of the image computed as explained in the theory section from reference [12]. As seen from this table, it is noted that the g-DENOISER technique has better edge definition than either of the two techniques applied independently.

## 5. CONCLUSION

In this work, we have presented an efficient denoising technique g-DENOISER with a fairly insensitive choice of parameters for a given acceleration factor and a very robust termination criterion which accounts for the noise levels in the input image, and some ways to evaluate the image quality. This technique can be applied to improve the quality of images acquired using SENSE with high reduction factors. Work is in progress to evaluate the robustness of this algorithm at different reduction factors for multiple applications and to come up with a better image quality evaluation criteria.

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