

CORRECTION OF SOME TIME-DEPENDENT DEFORMATIONS IN PARALLEL-BEAM COMPUTED TOMOGRAPHY

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ABSTRACT

In computed tomography on interventional X-ray systems, image quality is frequently degraded by uncontrolled patient motion such as breath-hold failures, intestinal contractions, or nervous shaking. To overcome this problem, an iterative workflow is proposed to estimate a dynamic displacement field representing the time-varying position of image elements. An elastic signal registration algorithm computes the displacement in projection space from the difference between measured projections and reference projections, sampled from the image reconstructed in previous iterations. Considering the sampled image as a motionless reference, the motion estimation is exact for a certain class of deformations, including shifting, expansion, and compression. From a new estimate of the displacement field, a better image can be reconstructed by introducing motion compensation in the backprojection step of filtered-backprojection methods. The result of the first iteration is equivalent to a standard reconstruction without motion correction and further iterations progressively sharpen the image.

Index Terms— X-ray tomography, image reconstruction, motion estimation, iterative methods.

1. INTRODUCTION

A new trend in minimally invasive medical interventions is tomographic soft tissue imaging for diagnosis, therapy planning, and outcome control. The interventional room is typically equipped with a versatile digital radiography system mounted on a robotic C-arm. By programming a circular trajectory for the C-arm, X-ray transmission projections can be acquired at regular angular intervals around the patient. Using computed tomography (CT), a volumetric image can be reconstructed from the set of projections, unveiling the depth of anatomical structures.

During a treatment session, the patient will follow several acquisitions for which he is asked to retain a still position while holding his breath. An acquisition lasts for 10 to 20 seconds for an angular range of about 240 degrees. Unfortunately, uncontrolled patient movements yield inconsistent projections, which result in strong image artifacts when using analytical reconstruction methods. For example, such residual motion frequently occurs due to breath-hold failures, intestinal contractions, or nervous shaking.

The present work aims at estimating non-periodic motion from tomographic projections for high-quality static low contrast imaging on C-arm systems. This goal is related to more popular problems like the time-resolved dynamic cardiac reconstruction on closed gantry scanners [1], the reconstruction from free breathing acquisitions on

cone-beam systems mounted on radiotherapy linear accelerators [2, 3], or the motion extraction by tracking of fiducials [4].

In the following, only breath-hold (hence, non-gated) acquisitions are considered, for reconstruction of one single static image in which residual unstructured motion is estimated and compensated. Because previous efforts assume periodicity of the motion, the development of a different technical approach was required. The proposed estimation method relies solely on projections and does not require biological signal extraction or feature tracking.

The motion correction problem can be split into two complementary sub-problems: motion estimation and motion compensation. An analytical motion-compensated reconstruction algorithm has already been developed by Desbat, Roux, and Grangeat [5, 6]. The reconstruction is exact for the class of deformations that preserves some properties of the projection geometry. The present work provides a solution for the motion estimation problem in parallel-beam geometry and for a certain class of deformations, including shifting and some local expansions or compressions.

The remainder of this paper is structured as follow. First, an iterative motion correction framework is introduced. Then, an algorithm is derived to compute deformations while assuming the availability of a static reference image. A generalization allows estimating more general motion with a simple elastic signal registration method. Finally, results are shown for a randomly deformed Shepp-Logan phantom and conclusions are drawn.

2. MOTION CORRECTION WORKFLOW

In 2D tomography, projections are 1D signals and the projection of the displacement of image elements can be described by a smooth strictly increasing bijective mapping function in projection space. The strict increasing property appears because crossing of two integration lines never occurs with plausible deformations. In fan-beam geometry, the position of the focus point is parameterized by a projection angle and a distance from the detector. Due to relative motion, the focus position can vary freely over time, describing a so-called *virtual* source trajectory. Therefore, all line integrals are measured between a displaced focus point and pixels, smoothly displaced on the projection axis. The parallel-beam geometry considered here is a special case of the fan-beam geometry in which the focus point is at infinite distance from the projection plane. Therefore, only the projection angle parameter can vary.

If the virtual source trajectory matches the ideal circular path, the exact motion compensation of Desbat *et al.* only involves two slight modifications in the backprojection step of classical filtered backprojection (FBP) algorithms. First, the integration line joining the center of the current voxel to the focus point is displaced, according to the provided bijective mapping and the line integral value

Colas Schretter received financial support from the Marie Curie program of the European Union under contract MEST-CT-2005-020424.

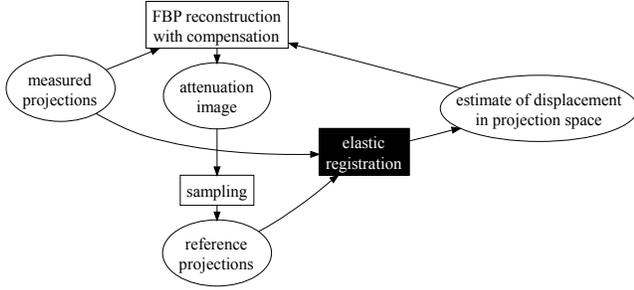


Fig. 1. Iterative workflow for motion estimation and compensation.

is fetched at the corresponding pixel in the projection image. Second, the line integral is weighted by a scaling coefficient, preserving the total mass in cases of local expansion or compression movements. The scaling coefficients are proportional to the derivative of the scalar mapping function.

The iterative motion correction workflow, outlined in Fig. 1, proposes a solution to estimate the mapping function. Data is represented by ovals and boxes symbolize algorithms. For each iteration, an *attenuation image* is reconstructed from the *measured projections* using the current *estimate of displacement in projection space*. Then, *reference projections* are sampled by computing the forward projection through the *attenuation image* reconstructed so far. The *measured projections* are mapped on the *reference projections* by an *elastic registration* algorithm. The resulting warping is a new *estimate of displacement in projection space* that can be used by the *FBP reconstruction with compensation* to produce a new, refined, *attenuation image*.

The first reconstruction assumes no motion and the resulting image contains artifacts, but may nevertheless be considered as a motionless reference for the second iteration. Since these reference projections are sampled from a static image, they are consistent. Therefore, the registration algorithm will provide a displacement field in projection space that compensates for inconsistencies when reconstructing an image from measured projections.

3. MOTION ESTIMATION

Let's assume the availability of a motionless image from which reference projections can be sampled. The considered motion estimation problem is to extract the displacement of pixels in projection space from corresponding measured and reference projections. The motion is computed independently for each pair of projections.

3.1. Notations

A static 2D image is commonly represented by a Cartesian grid of point samples, located at centers of identical image elements: usually, non-overlapping square pixels. It is easy to extend this image model to represent motion by associating, for each pixel, a description of its displacement over time from the initial grid position. In a similar way the space is discretized by the grid, the displacement of pixels can be sampled at several time frames. In this paper, one time frame is associated with each projection.

Let $f(x, y, t)$, a dynamic 2D image where (x, y) are spatial Cartesian coordinates and $t \in [0, 1)$ is the normalized temporal position. The function f is positive and compactly supported in a normalized circular field of view such that $f(x, y, t) = 0$ when

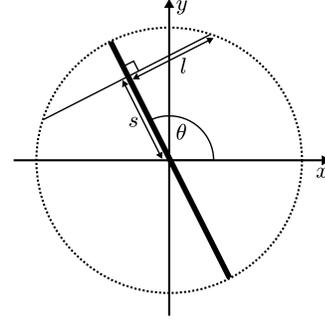


Fig. 2. Parameterization of line integrals for parallel-beam geometry.

$x^2 + y^2 > 1$. Moreover, the total mass M must be preserved:

$$M = \int \int f(x, y, t) dx dy, \forall t.$$

As shown in Fig. 2, a line integral $p(\theta, s, t)$ from f is parameterized by an angular coefficient $\theta \in [0, \pi)$ and a signed distance from the origin $s \in [-1, 1]$ and is defined by

$$p(\theta, s, t) = \int_{-l}^l f(s \cos \theta + u \sin \theta, s \sin \theta - u \cos \theta, t) du,$$

where $l = \sqrt{1 - s^2}$ is the half-length of the intersection between the line and the field of view bounded by the dotted circle. The thick line segment shows the orientation of the virtual detector which passes through the origin and is orthogonal to the projection direction. The point of intersection between the line and the virtual detector is $(s \cos \theta, s \sin \theta)$ and the normalized direction vector of the integration line is $(\sin \theta, -\cos \theta)$.

The Radon transform of f is the collection of all time-varying line integrals intersecting the field of view. The Radon transform provides sufficient data to reconstruct exactly the dynamic image f at any position in space and time. However, most CT tomographs are only able to measure line integrals along one projection direction at a time. To model this limitation, the projection angle θ is assumed to be linearly dependent on the acquisition time t . Therefore, $\theta = \pi t$ such that one half circular rotation is achieved when $t = 1$. In the following, the Radon transform is defined as the measured projections

$$(Rf)(\theta, s) = p\left(\theta, s, \frac{\theta}{\pi}\right),$$

and this collection of line integrals could be inconsistent since each projection observes f at a potentially different deformation state.

3.2. Shifting Motion

The *translation invariance* property of the Radon transform states that translation in the image domain results in shifted projections. This important property and its relation to image motion has been studied further by Milanfar [7]. An image displaced by the translation vector (d_x, d_y) is noted

$$f_d(x, y, t) = f(x + d_x, y + d_y, t),$$

and its Radon transform is obtained by translating each projection:

$$(Rf_d)(\theta, s) = (Rf)(\theta, s + d_x \cos \theta + d_y \sin \theta).$$

Therefore, analytical estimation of shifting motion is straightforward by computing a feature point that depends on global translation in both the measured projection and the reference projection. For instance, the center of mass

$$\mu(\theta) = \frac{1}{M} \int_{-1}^1 s (Rf)(\theta, s) ds$$

can be computed in projection space from the weighted mean of pixel positions on the virtual detector axis. The difference between center of masses in corresponding measured and reference projections yields a signed shifting displacement.

3.3. Global Expansion or Compression Motion

Given the *scaling invariance* and the *linearity* properties of the Radon transform, global expansion or compression in the image domain results on respectively broader or narrower spreads in projection space, while preserving the total mass. An image scaled non-uniformly along the horizontal and vertical axis by the respective factors $s_x > 0$ and $s_y > 0$, and preserving the total mass is noted

$$f_s(x, y, t) = \frac{1}{s_x s_y} f\left(\frac{x}{s_x}, \frac{y}{s_y}, t\right),$$

and its Radon transform is obtained by weighting each projection, translated toward or away from the origin:

$$(Rf_s)(\theta, s) = \frac{1}{\alpha} (Rf)(\theta, s\alpha),$$

where $\alpha = \sqrt{(s_x \cos \theta)^2 + (s_y \sin \theta)^2}$.

Therefore, analytical estimation of global expansion and compression deformations is also straightforward by computing a value that depends on the scale of corresponding projections. For instance, the standard deviation from the center of mass

$$\sigma(\theta) = \sqrt{\frac{1}{M} \int_{-1}^1 (s - \mu(\theta))^2 (Rf)(\theta, s) ds}$$

can be computed for both measured and reference projections. The ratio of the former on the later yields the scaling factor.

3.4. Shifting and Local Expansion or Compression Motion

As explained above, analytical extraction of motion in projection space is trivial for cases such as shifting and global expansion or compression. The center of mass can be computed in projection space from the weighted mean of pixel positions, in both measured and reference projections. Their difference gives the projection of the shifting displacement and the ratio of standard deviations gives the object scale, as perceived from the projection.

Let N , the number of bins in a projection histogram. The cumulative sum of bins is invariant in respect to the projection angle and is equal to M . Also, the point located at the center of mass splits the histogram in two parts of equal partial integrals. Generalizing this observation, any particular point

$$p_i = \frac{2i - N - 1}{N}, i \in [1 \dots N]$$

of a projection splits the signal in two regions determined by the values of their partial integrals. Systematic computation of the correspondence between centers of pixels defines a discrete bijective mapping function that registers two discrete signals.

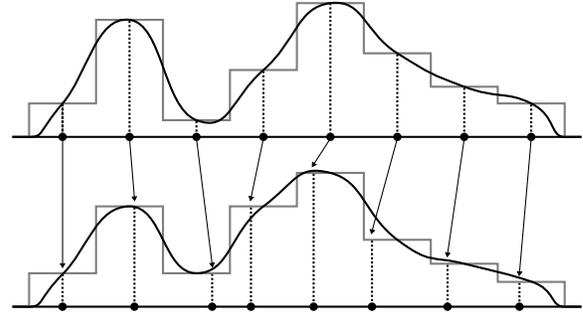


Fig. 3. Elastic signal registration.

As shown in Fig. 3, the center of every pixels p_i of the source histogram is mapped on sub-pixel accurate locations $q_i \in [-1, 1]$ on the target histogram, such that partial integrals are preserved:

$$\int_{-1}^{p_i} (Rf)(\theta, s) ds = \int_{-1}^{q_i} (Rf_{ref})(\theta, s) ds,$$

where f_{ref} is the motionless reference image. One can remark that $q_i \leq q_j, \forall i < j$ is an invariant of the former relation, ensuring that the computed discrete function is a strictly increasing bijective mapping. Offsets between corresponding pixels are displacements in projection space.

This elastic signal registration procedure can be implemented by a simple numerical integration method that marches through sub-pixel locations in the target histogram to find the correspondence with the centers of pixels from the source histogram. Two corresponding points share identical value for their left and right partial integrals. Elastic image registration algorithms minimize, usually iteratively, an image dissimilarity metric. In this case, the dissimilarity is the difference between partial integrals in the source and target histograms and the solution is not iterative.

4. RESULTS

Results from an experiment with the low-contrast Shepp-Logan phantom are shown in Fig. 4 and Fig. 5, for random shifting and non-uniform global compression motion. Measured data has been simulated for a full rotational acquisition in parallel-beam geometry. From 256 projections of 128 pixels, images of 128×128 pixels are reconstructed.

Fig. 4 shows the projections and reconstructed images for the first steps within the iterative motion correction technique explained in Fig 1. The reconstructed images are shown with 2×2 magnification of pixels size. An efficient correction for the inconsistencies of input projections is already observed after the second iteration.

Fig. 5 shows the estimated displacement of pixels in projection space for successive iterations. Bright and dark pixels correspond to positive and negative offsets, respectively. Residual errors decrease with increasing iteration number and are mainly located at the edge of the object, at the border between high and low attenuation regions. Each row is filled with a gradient: from bright to dark for compressions and from dark to bright for expansions. A constant negative or positive bias contains the global shifting information. There is no correlation between rows because the deformations are generated randomly over time. More complicated motion could yield complex non-linear variations along the horizontal axis.

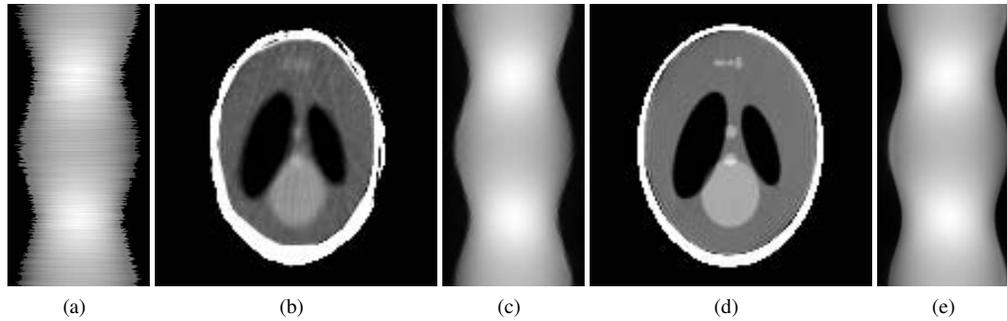


Fig. 4. Motion correction results. Reconstructions from simulated measured projections (a) of a randomly shifted and compressed phantom are shown after the first (b) and second (d) iteration. The projections (a) are inconsistent and regular FBP reconstruction leads to a motion-corrupted image (b). Forward projections from (b) give the reference projections (c). A displacement field is estimated in projection space (result shown in Fig. 5) by elastic registration of measured projections (a) to reference projections (c). Using the displacement for a motion-compensated FBP reconstruction from measured projections (a) yields the motion corrected image (d) from which more accurate reference projections (e) can be sampled.

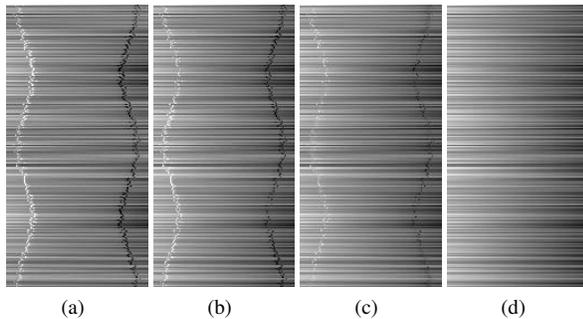


Fig. 5. Motion estimation results. Images show the estimates of displacements in projection space after the first (a), second (b), and third (c) iteration. The last pattern (d) is the ground truth.

5. CONCLUSION

Motion correction is a crucial problem, especially for slowly rotating CT, and recent releases in literature indicate that this topic becomes increasingly popular. However, the estimation of non-periodic residual motion in breath-hold acquisitions has not been tackled so far.

This paper introduces a general methodology for motion estimation and compensation in tomography. In particular, a solution is provided to estimate the motion information for the class of deformations that can be described by a strictly increasing bijective mapping function in projection space. Tackling these specific deformations is inspired by the complementary work of Desbat *et al.*, showing that exact reconstruction from inconsistent projections and a motion description, is still possible for this class of deformations.

The extraction of the motion information is based on numerical integration, without using any prior knowledge about the temporal or spatial smoothness of the underlying displacement field. This is a simple non-iterative elastic signal registration procedure that can be computed in a single pass over the input data.

The proposed iterative motion correction framework can be interpreted as an expectation-maximization (EM) method. The E step is the sampling of reference projections and the computation of a displacement field by elastic registration to the corresponding mea-

sured projections. The M step is the analytical motion-compensated reconstruction that results in a new reference image. If no motion corrupts the projections, the process converges in one iteration and reduces to a standard image reconstruction.

The methodology can be relevant for a number of X-ray-based modalities such as C-arms and radiotherapy systems. An example of a specific application is the correction of residual motion for soft tissue imaging from breath-hold acquisitions. Future investigations will extend the class of admissible deformations and apply the motion correction technique to the fan-beam geometry. A similar solution could also be proposed for the 3D cone-beam geometry. Although it is expected that, for 2D projections, the image registration algorithm has to be very different than the one presented here.

6. REFERENCES

- [1] K. Taguchi, W. P. Segars, E. K. Fishman, and B. M. W. Tsui, "Image-domain motion compensated time resolved 4D cardiac CT," in *SPIE Medical Imaging*, vol. 6510, 2007.
- [2] J.-J. Sonke, L. Zijp, P. Remeijer, and M. van Herk, "Respiratory correlated cone beam CT," *Medical Physics*, vol. 32, no. 4, pp. 1176–1186, 2005.
- [3] R. Zeng, J. A. Fessler, and J. M. Balter, "Estimating 3-D respiratory motion from orbiting views by tomographic image registration," *IEEE Trans. Medical Imaging*, vol. 26, no. 2, pp. 153–163, 2007.
- [4] W. Lu and T. R. Mackie, "Tomographic motion detection and correction directly in sinogram space," *Phys. Med. Biol.*, vol. 47, no. 8, pp. 1267–1284, 2002.
- [5] S. Roux, L. Desbat, A. Koenig, and P. Grangeat, "Exact reconstruction in 2D dynamic CT: compensation of time-dependent affine deformations," *Phys. Med. Biol.*, vol. 49, no. 11, pp. 2169–2182, 2004.
- [6] L. Desbat, S. Roux, and P. Grangeat, "Compensation of some time dependent deformations in tomography," *IEEE Trans. Medical Imaging*, vol. 26, no. 2, pp. 261–269, 2007.
- [7] P. Milanfar, "A model of the effect of image motion in the radon transform domain," *IEEE Trans. Image Processing*, vol. 8, no. 9, pp. 1276–1281, 1999.