

CONSTRUCTION OF ENDOCARDIAL AND EPICARDIAL SURFACE MODELS FROM SEGMENTED MRI

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ABSTRACT

This paper presents a novel method for the construction of endocardial and epicardial surface models from 3D short-axis cardiac magnetic resonance images with strongly anisotropic voxels in the long-axis direction. The same algorithm is independently used to generate the surface meshes of the epicardium and endocardium of the four cardiac chambers. The proposed method provides smooth meshes of the heart chambers despite the strong voxel anisotropy. This is not the case for the marching cubes algorithm. Furthermore, the presented method generates more regular mesh triangles than the marching cubes and allows for a complete control of the number of triangles. However, the generated surface meshes are still close to the ones obtained by the marching cubes. For the five tested cases, the average distance between the surfaces generated by our method and by the marching cubes algorithm was 0.4 mm.

Index Terms— Surface generation, Cardiac MRI.

1. INTRODUCTION

Surface models of the heart are used for visualization and modeling (e.g. FEM) purposes. A popular approach for constructing such models is to apply the marching cubes algorithm [1] to segmented cardiac magnetic resonance image (MRI). However, cardiac MRI typically has strongly anisotropic voxels. The voxel size in one direction is usually at least five times larger than in the other two directions. A direct consequence of the voxel anisotropy is that the marching cubes algorithm generates a mesh with pronounced steps and elongated triangles, as shown in Fig. 1. The mesh obtained by the marching cubes can be significantly smoothed. However, the smoothing of the mesh shrinks it, which makes it a less accurate representation of the chamber surface. Here, we present a new method for the generation of a four-chamber surface model from segmented cardiac MRI, which generates smooth meshes of the heart chambers despite the strong voxel anisotropy of the 3D images.

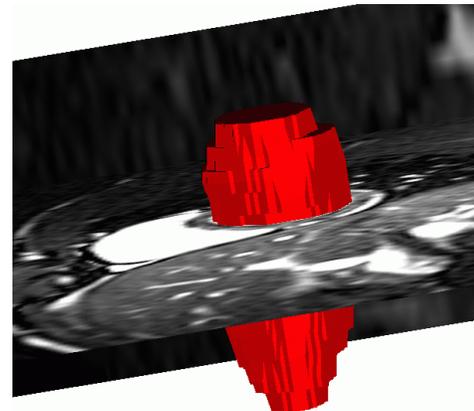


Fig. 1. A left ventricular surface model generated by applying the marching cubes algorithm to segmented cardiac MRI.

2. METHODS

2.1. Motivation

The input to the method is the segmentation of each of the four cardiac chambers. The same algorithm is independently used to generate the surface mesh of the epicardium and of the endocardium of the four cardiac chambers. A number of algorithms have been designed to construct a regular triangulated surface mesh on a sphere [2]. The number of vertices and triangles of the mesh can be specified. The main idea of the method is to propagate in a one-to-one and continuous (i.e. homeomorphic) manner the mesh from the sphere to the cardiac chamber. A mesh is constructed on a sphere that is centered at the barycenter of the chamber and includes the chamber. We use an approach based on the Laplace equation to propagate the mesh from the sphere to the cardiac chamber for the following reasons. If we consider a flow field \mathbf{v} transporting particles between two surfaces A and B , the homeomorphic condition requires a potential flow, i.e. the flow field can be expressed as the gradient of a scalar potential u , where $\mathbf{v} = \nabla u$. If the medium is assumed to be incompressible, the

flow field v must be divergence free, i.e. $\nabla \mathbf{v} = 0$. This condition leads to $\Delta u = 0$, which is the Laplace equation. Solving the Laplace equation between the surfaces A and B will lead to a one-to-one correspondence between the two surfaces following the direction of the gradient of u . Some research efforts have also used the Laplace equation for colon surface flattening [3], thickness measurement [4, 5], and local shape variability measurement [6].

2.2. Mapping of the Sphere to the Cardiac Chamber Surface

2.2.1. Harmonic function with a spherical isolevel and a single singularity

We first derive the solution to the Laplace equation over a spherical domain that has a singularity somewhere within the domain and that is equal to a constant on the boundary of the domain. Let the sphere center be the coordinate system origin, \mathbf{R} denotes the radius of the sphere, \mathbf{s} the location of the singularity, and \mathbf{r} the independent variable. The solution $f_s(\mathbf{r})$ needs to satisfy the following:

$$\lim_{\mathbf{r} \rightarrow \mathbf{s}} f_s(\mathbf{r}) = \infty, \quad (1)$$

$$f_s(\mathbf{r})|_{|\mathbf{r}|=\mathbf{R}} = 0, \quad (2)$$

$$\Delta f_s = 0. \quad (3)$$

The fundamental solution of the Laplace equation in 3D is $\frac{1}{|\mathbf{r}|}$ and it represents a singularity at the origin. The fundamental solution centered at \mathbf{s} , i.e. function $\frac{1}{|\mathbf{r}-\mathbf{s}|}$, satisfies Eqs. 1 and 3 but it does not satisfy the boundary condition Eq. 2. It turns out that the sum of two shifted fundamental solutions, one centered at \mathbf{s} and one centered at $\frac{\mathbf{s}\mathbf{R}^2}{|\mathbf{s}|^2}$ and multiplied by $-\frac{\mathbf{R}}{|\mathbf{s}|}$, satisfy Eqs. 1, 2, and 3. Note that the second singularity is outside the sphere, i.e. there is only one singularity within the spherical domain. The solution is:

$$f_s(\mathbf{r}) = \frac{1}{|\mathbf{r}-\mathbf{s}|} - \frac{\mathbf{R}}{|\mathbf{s}|} \frac{1}{|\mathbf{r}-\frac{\mathbf{s}\mathbf{R}^2}{|\mathbf{s}|^2}|}. \quad (4)$$

It is straightforward to show that Eq. 4 satisfies Eqs. 1, 2, and 3. Also $f_s(\mathbf{r}) > 0$ when $|\mathbf{r}| < \mathbf{R}$, $f_s(\mathbf{r}) = 0$ when $|\mathbf{r}| = \mathbf{R}$, and $f_s(\mathbf{r}) < 0$ when $|\mathbf{r}| > \mathbf{R}$.

2.2.2. Potential between the sphere and the cardiac chamber

Let \mathbf{s}_m , $m = 1, \dots, M$ represent the locations of M singularities. We represent the potential as a sum of the functions defined by Eq. 4:

$$u(\mathbf{r}) = c \sum_{m=1}^M f_{\mathbf{s}_m}(\mathbf{r}). \quad (5)$$

Note that $\Delta u = 0$, $u(\mathbf{r}) = 0$ on the sphere (i.e. when $|\mathbf{r}| = R$) and that there are M singularities centered at \mathbf{s}_m .

Let \mathbf{r}_n , $n = 1, \dots, N$ represent the boundary points of the cardiac chamber. The goal is to fit the boundary of the cardiac chamber to an isolevel surface equal to one. The fitting is performed in the least squares sense. Therefore the objective function to be minimized is:

$$O = \frac{1}{2} \sum_{n=1}^N [u(\mathbf{r}_n) - 1]^2. \quad (6)$$

The constant c is obtained by solving the equation $\frac{\partial O}{\partial c} = 0$. The solution is:

$$c = \frac{\sum_{n=1}^N d_n}{\sum_{n=1}^N d_n^2}, \quad (7)$$

where $d_n = \sum_{m=1}^M f_m(\mathbf{r}_n)$. Thus, the potential u defined in Eq. 5 satisfies the Laplace equation, has an isolevel equal to 0 on the sphere and an isolevel equal to 1 that is close to the boundary of the cardiac chamber.

2.2.3. Surface mesh of the cardiac chamber

We want to propagate the mesh vertices from the sphere (i.e. isolevel 0) to the boundary of the cardiac chamber along the gradient of u . Thus, we need to determine when to stop the propagation, which corresponds to finding the final value of the potential for each mesh vertices of the sphere. To do so, we first propagate the points of the chamber boundary \mathbf{r}_n to the sphere along the negative gradient of u by using the following partial differential equation:

$$\frac{\partial \mathbf{r}(t)}{\partial t} = -\nabla u(\mathbf{r}(t)), \quad (8)$$

$$\mathbf{r}(0) = \mathbf{r}_n.$$

We use the fourth-order Runge-Kutta numerical method [7] to propagate the points numerically. We stop the propagation when the points reach the sphere, i.e. when their potential become 0.

Then, for all the chamber boundary points, we compute their potential values $u(\mathbf{r}_n)$ and we assign them to the corresponding points on the sphere p_n obtained by the propagation. We use a pseudo-thin plate spline on the sphere [8] to interpolate those potential values and obtain the potential values on the cardiac chamber boundary $l(\mathbf{p})$:

$$l(\mathbf{p}) = \alpha + \sum_{k=1}^K \beta_k \Psi(\mathbf{p} \cdot \mathbf{q}_k), \quad (9)$$

where p is the point on the sphere where we want to obtain the potential value, K are the number of control points on the sphere used for the interpolation, q_k are the unit vectors of the control points from the center of the sphere, Ψ is a function given in [8], and α and β_k are constants to be computed. We determine the constants α and β_k such that the potential values of all the chamber boundary points $u(\mathbf{r}_n)$ are fitted to the least squares sense. Note that we want to use a relatively

small number of control points, i.e. the number of boundary points N is much larger than the number of control points K . The fitting of the chamber boundary points is expressed by:

$$l(\mathbf{p}_n) = u(\mathbf{r}_n), \quad (10)$$

which corresponds to the following matrix system:

$$\underbrace{\begin{bmatrix} 1 & \Psi(\mathbf{p}_1 \cdot \mathbf{q}_1) & \dots & \Psi(\mathbf{p}_1 \cdot \mathbf{q}_K) \\ 1 & \Psi(\mathbf{p}_2 \cdot \mathbf{q}_1) & \dots & \Psi(\mathbf{p}_2 \cdot \mathbf{q}_K) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \Psi(\mathbf{p}_N \cdot \mathbf{q}_1) & \dots & \Psi(\mathbf{p}_N \cdot \mathbf{q}_K) \end{bmatrix}}_G \underbrace{\begin{bmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_K \end{bmatrix}}_x = \underbrace{\begin{bmatrix} u(\mathbf{r}_1) \\ u(\mathbf{r}_2) \\ \vdots \\ u(\mathbf{r}_N) \end{bmatrix}}_h. \quad (11)$$

The corresponding least squares solution is:

$$x = (G^T G)^{-1} G^T h. \quad (12)$$

Thus, for any point on the sphere \mathbf{p} , we can obtain a potential value that corresponds to the potential limit on the cardiac chamber boundary $l(\mathbf{p})$ for the propagation along the gradient of u .

Finally, to propagate the mesh vertices \mathbf{p}_v from the sphere to the cardiac chamber boundary, we use the same propagation method given by Eq. 8 but in the opposite direction. We stop the propagation when we reach the corresponding potential values $l(\mathbf{p}_v)$.

3. RESULTS

The method has been tested on five different cases for all the chambers: left ventricle (LV), right ventricle (RV), left atrium (LA), right atrium (RA). The singularities are placed automatically inside the segmented chamber. Fig. 2 illustrates the singularity locations for RV. The number of singularities for each chamber is reported in Table 1. We use the same number of control points for the interpolation on the sphere as the number of singularities, i.e. $K = M$. Figs. 3 and 4 show the surface meshes of the cardiac chambers generated by the proposed method. Fig. 5 shows one slice of each of the segmented chamber with the corresponding surface meshes generated by the marching cubes algorithm and the proposed method.

Table 1 provides the average in-slice distance between the surfaces meshes generated by the marching cubes algorithm and the proposed method. Those distances are obtained by manual measurement for all the chambers. The average is performed over all the slices of each cardiac chamber for the five cases.

4. CONCLUSION

We developed a new method for constructing endocardial and epicardial surfaces from 3D segmented MRI. The same algorithm is applied independently to each cardiac chamber. A

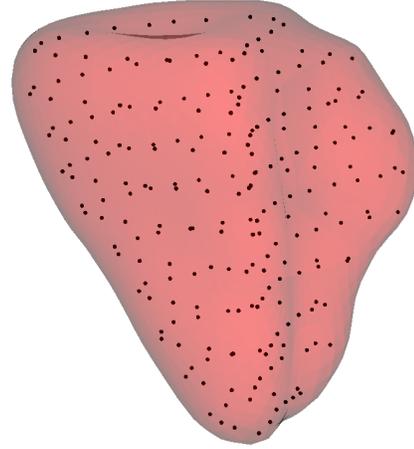


Fig. 2. Arrangement of singularities for the RV endocardium. The singularities are placed uniformly inside the segmented chamber.

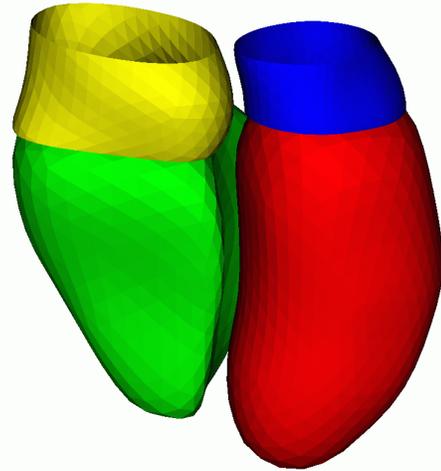


Fig. 3. Endocardium surface meshes generated by the proposed method for LV (red), RV (green), LA (blue), and RA (yellow).

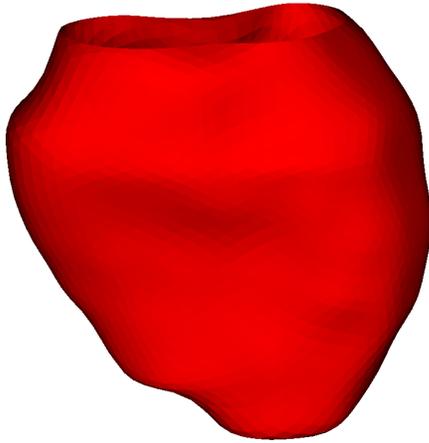


Fig. 4. Epicardium surface mesh generated by the proposed method for the entire myocardium.

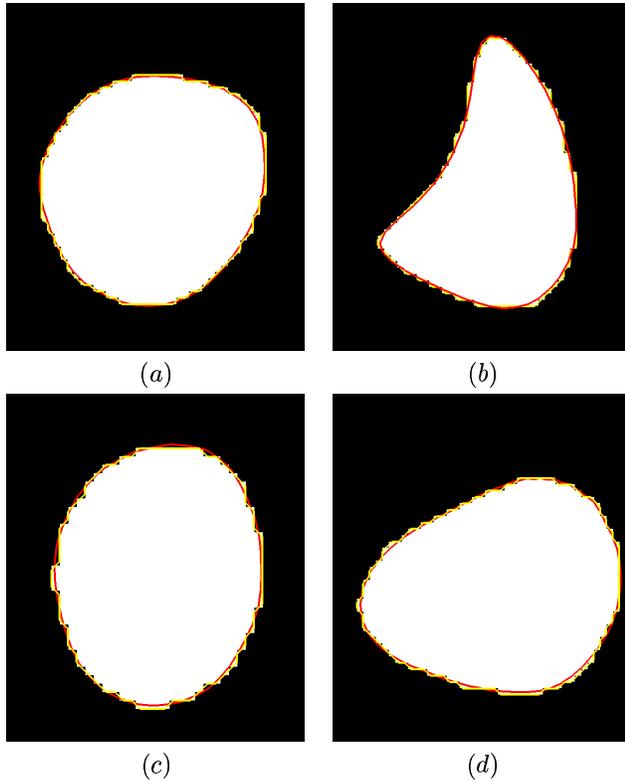


Fig. 5. Contours of the surface meshes generated by the marching cubes algorithm and the proposed method for (a) LV, (b) RV, (c) LA, and (d) RA. The red contours correspond to the proposed method and the yellow ones to the marching cubes.

Chamber	M	Distance to Marching Cubes [mm]
LV endocardium	222	0.4
RV endocardium	208	0.3
LA endocardium	55	0.5
RA endocardium	48	0.4
Myocardium epicardium	436	0.3

Table 1. This table provides for each chamber the average distance between the surfaces meshes generated by the marching cubes and the proposed method. The in-plane resolution is 1.44 mm x 1.44 mm.

surface mesh is first constructed on a sphere that includes the cardiac chamber. The mesh vertices are propagated along the gradient of a potential that satisfies the Laplace equation. The resulting surface meshes are smooth despite the strong voxel anisotropy, which is not the case for the marching cubes algorithm. Furthermore, the presented method generates more regular mesh triangles than the marching cubes and allows for a complete control of the number of triangles. For the five tested cases, the average distance between the surfaces generated by our method and by the marching cubes was 0.4 mm.

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