IMPROVED SPIRAL SENSE RECONSTRUCTION USING A MULTISCALE WAVELET MODEL

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ABSTRACT

SENSE has been widely accepted and extensively studied in the community of parallel MRI. Although many regularization approaches have been developed to address the ill-conditioning problem for Cartesian SENSE, fewer efforts have been made to address this problem when the sampling trajectory is non-Cartesian. For non-Cartesian SENSE using the iterative conjugate gradient method, ill-conditioning can degrade not only the signal-to-noise ratio, but also the convergence behavior. This paper proposes a regularization technique for non-Cartesian SENSE using a multiscale wavelet model. The technique models the desired image as a random field whose wavelet transform coefficients obey a generalized Gaussian distribution. The effectiveness of the proposed method has been validated by *in vivo* experiments.

Index Terms— Spiral SENSE, multiscale model-based regularization, wavelet transformation

1. INTRODUCTION

Parallel MRI is an advanced fast imaging method to reduce k-space data acquisition time without the need for improved gradient performance. Several reconstruction methods have been established in order to unfold the aliased images caused by sampling at a rate lower than Nyquist rate during acquisition [1,2]. Among them, SENSE (SENSitivity Encoding) [1] has been accepted as one of the standard reconstruction methods due to its theoretical optimality. In the basic SENSE reconstruction, the underlying inverse solution can be ill-conditioned to a degree that increases with reduction factor. Tikhonov regularization has been successfully and widely used to reduce the problem due, in part, to the existent of a closed form solution [3,4]. It can be easily applied to the direct matrix inversions used for uniform Cartesian trajectories [1] and the iterative conjugate gradient (CG) method for other trajectories [5]. However, Tikhonov regularization is known to blur sharp edges. Some recent work has applied edge-preserving regularization techniques to SENSE reconstruction [6,7]. Due to the piecewise smooth prior, the reconstruction can have blocky effects due to loss of detailed structures. There is a need to further improve the SENSE regularization.

In this paper, we propose a novel reconstruction method for non-Cartesian SENSE using a Bayesian approach. A critical challenge for Bayesian image reconstruction methods is the choice of prior models for images. Ideally, a good prior model should accurately reflect the smooth and textured regions as well as preserve edges of images. We model the images to be reconstructed as a random field whose wavelet coefficients obey a generalized Gaussian (GG) distribution [8-10]. It has been shown that such a multiscale stochastic prior model can satisfactorily represent both smooth and sharp features of images [8]. We apply this prior model to spiral SENSE reconstruction and demonstrate that the image quality of the proposed method is superior to that of the CG-SENSE with Tikhonov regularization.

2. FORMULATION OF SENSE RECONSTRUCTION

In SENSE, the imaging equation can be formulated as a linear operation of the transverse magnetization image [1,5]:

$$\mathbf{Ef} = \mathbf{d}, \tag{1}$$

where $\mathbf{d}$ is the vector formed from all k-space data acquired at all channels, and $\mathbf{f}$ is the unknown vector formed from the desired full field of view (FOV) image to be solved for, both with a lexicographical column ordering of the two-dimensional array components. The encoding matrix $\mathbf{E}$ consists of the product of Fourier encoding with subsampled k-space and coil-specific sensitivity modulation over the image, i.e.

$$\mathbf{E}_{\{l,m\},n} = e^{-i2\pi \vec{k}_m \cdot \vec{r}_n} s_l(\vec{r}_n), \tag{2}$$

where s_l denotes the sensitivity function of the lth channel, m and n denote the indices for the k-space data and image pixels, respectively.

In presence of additive Gaussian noise in MR data measurement [11], according to the imaging equation (1), $(\mathbf{d} - \mathbf{Ef})$ obeys a zero-mean Gaussian distribution whose covariance depends on noise correlations:

$$p(\mathbf{d} \mid \mathbf{f}) = \frac{1}{(2\pi)^{n/2} \mid \mathbf{\Psi} \mid^{1/2}} \exp[-\frac{1}{2}(\mathbf{d} - \mathbf{Ef})^H \mathbf{\Psi}^{-1} (\mathbf{d} - \mathbf{Ef})]. \tag{3}$$

SENSE reconstruction obtains the image $\mathbf{f}$ using the maximum likelihood estimation.

3. PROPOSED MULTISCALE MODEL-BASED SENSE RECONSTRUCTION METHOD

3.1. Bayesian Formulation

In Bayesian framework, the maximum a posteriori (MAP) estimation is used to incorporate the prior information when the statistics of the parameters to be estimated is given. In this case, the MAP estimation of the image is given by [12]:

$$\mathbf{f}_{\text{MAP}} = \arg\max_{\mathbf{f}} p(\mathbf{f}\,|\,\mathbf{d}) ,$$
$$= \arg\max_{\mathbf{f}} p(\mathbf{d}\,|\,\mathbf{f})p(\mathbf{f}) , \qquad (4)$$

where $p(\mathbf{f})$ is the prior representing the random field of the image. The accuracy of the prior in modeling the desired image is crucial in MAP estimation --- the additional information can improve the image quality but any bias may lead to image artifacts.

2.2. Multiscale Wavelet Model

We use a multiscale wavelet model for image prior,

$$p(\mathbf{f}) \propto \prod_{m,n} p(w_j^{(i)}(m,n)) \qquad (5)$$

where $w_j^{(i)}(m,n)$ is the (*m,n*)th coefficients of the wavelet transform of the image with superscript (i) being 0 when (m,n) is the scaling coefficient and 1, 2, or 3 when (m,n) is the wavelet coefficient at the vertical, horizontal, or diagonal orientations, respectively, and subscript j denoting the scale. Figure 1 (a) shows the coefficients image after 3-level wavelet decomposition. These coefficients are assumed to take a generalized Gaussian (GG) distribution [9, 10] as

$$P(w_j^{(i)}(m,n)) \propto \exp\left\{ -\frac{1}{p}\left|\frac{w_j^{(i)}(m,n)}{k_j^{(i)}}\right|^p \right\} \qquad (6)$$

where the power $0 < p \le 2$ is a parameter that determines the tail behavior of the density function and $k_j^{(i)}$ is a scale parameter similar to the standard deviation of a Gaussian density. We denote zero mean distribution as $GG(0, k_j^{(i)}, p)$. Equation (6) is the Laplacian density for p = 1, and the Gaussian density for p = 2. The parameter $k_j^{(i)}$ varies for each scale and wavelet coefficients in theory, but makes the model too complex to be useful in practice. Fortunately, the models proposed in Ref. [8] greatly simplify the choice of the parameter $k_j^{(i)}$, and make it possible to analyze the scale-to-scale structures and the orientation-dependent structure of images (see [8] for details). We use the first model in [8], where the scaling coefficients $w_{j0}^{(0)}(m,n)$ are i.i.d. (Independent and identically-distributed) with $GG(0, k_{j0}^{(0)}, p)$ and the wavelet coefficients are i.i.d. with exponentially decreasing variances, i.e., $w_j^{(i)}(m,n) \sim GG(0, k2^{-\alpha(j,j0)}, p)$, i = 1, 2, 3, and $j_0 \le j \le J-1$ with $j_0 \ge 0$ and $\alpha \ge 0$. This model is equivalent to a deterministic modeling of the image as a member of a Besov space. There are only two independent parameters in this model, which are chosen to be $\alpha = 1.2, p = 1$ as suggested in Ref. [8].

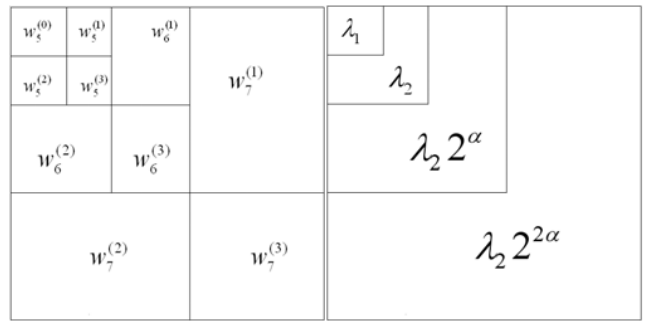

Fig. 1: (a) Image of multiscale coefficients and (b) the corresponding regularization parameters after 3-level wavelet decomposition.

2.3. Multiscale SENSE Reconstruction Algorithm

Plugging the multiscale wavelet prior into Eq. (4), the MAP estimation becomes the regularized reconstruction given as

$$\mathbf{f}_{\text{reg}} = \arg\min_f \left(\| \mathbf{d} - \mathbf{Ef} \|_2 + \left\| \mathbf{\Lambda W f} \right\|_p^p \right) \qquad (7)$$

where $\mathbf{W}$ denotes the wavelet transform matrix, Λ is a diagonal matrix whose elements are regularization parameters at different scales and orientations, and $\|\cdot\|_p$ denotes the Lp norm. Equivalently, the equation can be rewritten as

$$\mathbf{f}_{\text{reg}} = \mathbf{W}^T \left[\arg\min_w \left(\left\|\mathbf{d} - \mathbf{Hw}\right\|_2^2 + \lambda_{j_0}^{(0)} \left\| w_{j_0}^{(0)} \right\|_p^p + \sum_{j=j_0}^{J-1} \sum_{i=1}^{3} \lambda_j^{(i)} \left\| w_j^{(i)} \right\|_p^p \right) \right] \qquad (8)$$

where $\mathbf{H} = \mathbf{EW}^T$, and the regularization parameters from the diagonals of Λ are $\lambda_j^{(i)} = (2\sigma^2 / p(k_j^{(i)})^p)$ with $\lambda_{j_0}^{(0)} = \lambda_1$ $\lambda_j^{(i)} = \lambda_2 2^{\alpha(j-j_0)}$, and λ_1, λ_2, and α being constants.

3.2. Implementation

When the L_p-norm terms in Eq. (8) can not be differentiated, we have to resort to the smooth approximation to the p-norm as the following

$$\|w\|_p^p \approx \sum_{m,n} \left[(\left|w_{m,n}\right|^2 + \beta)^{p/2} - \beta^{p/2} \right] \qquad (9)$$

where $\beta \ge 0$ is a stabilization constant and $w_{m,n}$ denotes the element of the vector $\mathbf{w}$. We use lagged diffusivity fixed point algorithm [13] for the optimization of Eq. (8). Defining the objective function in Eq. (8) as

$$J = \left\|\mathbf{d} - \mathbf{Hw}\right\|_2^2 + \lambda_{j_0}^{(0)} \left\| w_{j_0}^{(0)} \right\|_p^p + \sum_{j=j_0}^{J-1} \sum_{i=1}^{3} \lambda_j^{(i)} \left\| w_j^{(i)} \right\|_p^p , \qquad (10)$$

the gradient $\nabla(\cdot)$ and Hessian $\nabla^2(\cdot)$ of the objective function to be minimized are given as

$$\nabla_J = \mathbf{H}^H\mathbf{Hw} - \mathbf{H}^H\mathbf{d} + \Lambda\mathbf{L}(\mathbf{w})\mathbf{w}, \quad (11)$$

and

$$\nabla^2_J \approx \mathbf{H}^H\mathbf{H} + \Lambda\mathbf{L}(\mathbf{w}) \quad (12)$$

where

$$\mathbf{L}(\mathbf{w}) = \mathrm{diag}\left[(|w_{m,n}|^2 + \beta)^{p/2-1}\right] \quad (13)$$

is a diagonal matrix with elements being the value in the bracket. The fixed point algorithm is given as follows.

Algorithm

$\mathbf{w}_0 =$ initial image

for i = 0,1, 2, ..., N {

$\mathbf{L}_i = \mathbf{L}(\mathbf{w}_i)$;

$\mathbf{g}_i = \mathbf{H^H H w}_i - \mathbf{H^H d} + p\mathbf{L}_i\mathbf{w}_i$; % gradient

$\mathbf{H}_i = \mathbf{H^H H} + p\mathbf{L}_i$; % approximation Hessian

$\mathbf{s}_i = -\mathbf{H}_i^{-1}\mathbf{g}_i$; % quasi-Newton step

$\mathbf{w}_{i+1} = \mathbf{w}_i + \mathbf{s}_i$; % update approximate solution

}

The quasi-Newton step in the fixed-point algorithm can be implemented using CG iterations. Specifically, each fixed-point iteration includes several inner CG iterations. To speed up the computation, nonuniform fast Fourier transform (NUFFT) [14] has been employed to calculate the sensitivity encoding for a non-Cartesian trajectory.

4. EXPERIMENT RESULTS

In vivo human spiral data have been acquired on a GE 3T scanner (GRE sequence, coil Num = 8, TE = 3.5ms, TR = 2s, flip angle = 90, FOV = 24cm, slice Thickness = 4.0mm, spacing = 1.0mm, matrix size = 256x256, slice num = 8, interleave num = 24, repetition = 10, spiral out). To simulate the reduced acquisition in parallel imaging, we manually selected 3, 4, 6, 8 out of 24 interleaves, corresponding to reduction factor R= 8, 6, 4, 3, respectively. In our implementation, we used p=1 (L_1 norm), $\alpha = 1.2$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, and $\beta = 0.01$. We show the result for R = 4 in Fig. 2(c) after 40 iterations. For comparison, we also show the sum of square (SoS) reconstruction using full data as well as the CG SENSE reconstruction for R = 4 [5] after 20 iterations (corresponding to the best result as showed in Fig. 3) in Fig. 2(a) and (b), respectively. In addition, we also show the result from CG SENSE with Tikhonov regularization [4] after 30 iteration in Fig. 2(d), which tends to smooth out fine structures of the image. Each iteration for the proposed method takes 4.2 seconds on a 2.8GHz CPU/512MB RAM PC, which is about the same as the running time of CG and regularized CG.

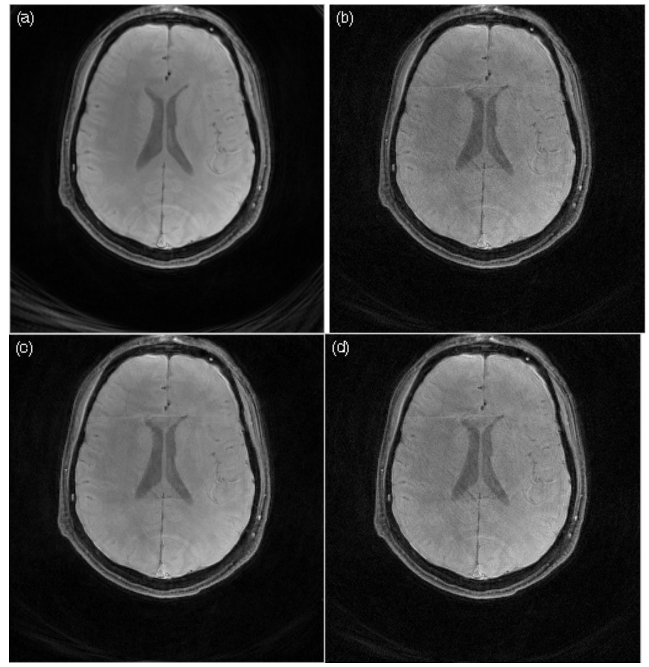

Fig. 2: SENSE reconstruction from in vivo data acquired with an 8- channel head coil and a 4x reduction factor. (a) SoS, (b) CG SENSE after 20 iteration, (c) the proposed method after 40 iterations and (d) Tikhonov regularization after 30 iterations.

We also compared the convergence curves of the proposed and the CG methods in Fig. 3 where the normalized mean squared errors (NMSE) were plotted as a function of the number of iterations at different reduction factors. It is seen that both the proposed method and the Tikhonov regularization give strictly deceasing NMSEs, but the former converges to lower errors. The results suggest that the proposed method can effectively addressed the so-called semi-convergence problem associated with CG approach [15], and is superior to Tikhonov regularization especially advantageous when the reduction factor is high.

5. DISCUSSION

The proposed method has some desirable features. The multiscale transformation provides freedom to manipulate certain frequency components of the image as Eq. (8) or (10) suggests. Although it is at the cost of complicated choice of regularization parameters, the model used here has only five independent parameters. The underlying optimization problem can be efficiently solved using the fixed-point algorithm, which is easy to implement and robust compared with other numerical method like primal-dual interior point solver [16]. The proposed method was applied to the spiral trajectory in this work, but is also applicable to arbitrary trajectories. When the parameter p is equal to 1, the proposed method can be regarded as an approximation of compressed sensing [17], where the spiral encoding matrix is a compressed sensing matrix and wavelet decomposition is a sparse representation.

In some specific applications, Model II or III in [8] might be preferred when the variance of the wavelet coefficients at different scales cannot be well-approximated by a simple exponential law, or when the image structures are orientation-dependent. Choice of model parameters will be further investigated in our future study.

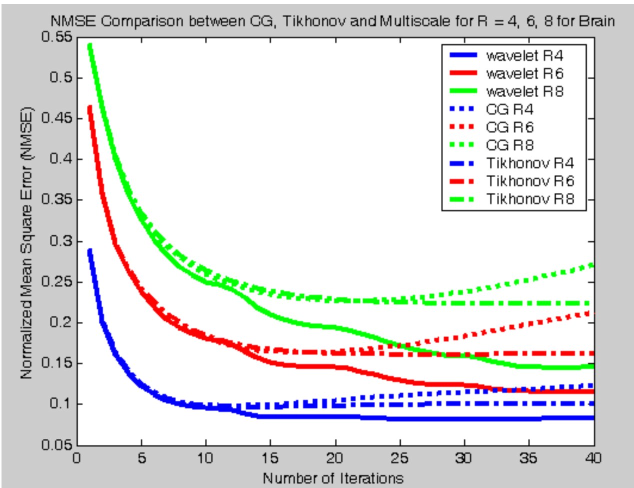

Fig. 3: Plots of NMSE vs. the number of iteration for CG (dashed lines) and the proposed multiscale model-based method (solid lines) at different reduction factors.

6. CONCLUSION

A novel method has been proposed to address the ill-conditioning problem in spiral SENSE reconstruction. Based on a multiscale wavelet model, the method is able to improve the convergence behavior of SENSE reconstruction while still maintaining sharp features. Results show that the algorithm achieves image quality superior to the existing CG method without extra computational burden.

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