

# HIGH DIMENSIONAL STATISTICAL SHAPE MODEL FOR MEDICAL IMAGE ANALYSIS

Heng Huang, Fillia Makedon

Computer Science and Engineering  
University of Texas at Arlington  
Arlington, TX, 76019

Roderick McColl

Radiology Department  
University of Texas Southwestern Medical Center  
Dallas, TX, 75390

## ABSTRACT

Statistical shape models have been widely used in biomedical image analysis, *e.g.* segmentation, registration, and shape classification. The traditional statistical shape models forced all shape parameters of each shape into one vector and put all vectors together to generate the point distribution model (PDM). The standard principal component analysis (PCA) was employed to project all shapes onto subspaces for dimensionality reduction. Since the shape vectors have a large dimension, the previous methods is computational expensive. In this paper, we propose a novel statistical shape models using natural PDM representations by multiple matrices and two dimensional PCA (2DPCA) is used to reduce the dimensionality of shape parameters. Because 2DPCA considers the correlations of row by row and column by column, our technique can fast extract the principle shape parameters. Combining with spherical harmonics shape representation, we create a framework for biomedical anatomic structures' shape analysis and classification. The experimental results using real cardiac left ventricle shapes have demonstrated our method outperforms the previous statistical shape modeling.

**Index Terms**— Statistical Shape Modeling, Shape Classification, PCA, 2DPCA, Cardiac Shape Classification

## 1. INTRODUCTION

Statistical shape models have proven to be useful tools to study variation in anatomical shapes. A popular method captures shape by a sampled 3D point distribution model (PDM) [1, 2]. The basic idea is to establish, from a training set, the pattern of legal variation in the shapes and spatial relationships of structures in a given class of images. Statistical analysis is used to give an efficient parameterization of this variability, providing a compact representation of shape. Statistical shape models consist of two major steps: (1) shape representation for extracting shape parameters; and (2) statistical analysis or pattern classification (learning a classifier) based on those parameters. In the previous statistical shape models methods, the standard principal component analysis (PCA) or related approaches [1, 2, 3] are used to reduce the dimensionality of shape parameters, because we usually have

much larger number of shape parameters (thousands) than the number of samples (hundreds or less). Since the PCA only can be performed onto vectors, the previous methods put all shape parameters of each shape into one vector and generate a matrix using  $N$  sample shapes. This is not a natural presentation for shape parameters. Recently, Yang *et al.* [4] proposed a two dimensional PCA (2DPCA) in which image covariance matrices are constructed directly using original image matrices and one-side low-rank approximation is applied. Based on 2DPCA, we propose a novel statistical shape models method without treating shape parameters of every shape as one vector (using one matrix to represent them). Compared to previous method, the new approach captures correlations between points by matrices based representations that reduce much more computational time. As the result, the performance of statistical shape models is also improved if we use the same storage and memory size.

Numerous 3D shape representation techniques have been proposed in the areas of medical image analysis, such as, landmark-based descriptors, deformation fields generated by mapping a segmented template image to individuals, distance transforms, medial axes, and parametric surfaces. This paper focuses on parametric surfaces using spherical harmonics. The use of surface harmonics for rigid and nonrigid shape description is well known. In previous work, Chen *et al.* presented motion and shape modeling primitives for the left ventricle [7]. Matheny and Goldgof [6] used 3D and 4D surface harmonics to reconstruct rigid and nonrigid shapes. Because they used the radial surface function ( $r(\theta, \phi)$ ) in all models, their methods are limited to represent only star-shape or convex objects without holes. However, this assumption is not true for many anatomic structures. *E.g.* the heart and its chambers are not actually star-shaped, especially as there are papillary muscles on the LV. Brechbühler *et al.* [8] presented an extended spherical harmonic (SPHARM) method to model any simply connected 3D object. A closed input object surface is assumed to be defined by a square surface parameter mesh converted from an isotropic voxel representation. The key component of this method is the mapping of surfaces of volumetric objects to parameterized surfaces prior to expansion into harmonics. SPHARM method have been applied in

many medical imaging applications, *e.g.*, shape analysis of brain structures [9, 5], cardiac functional measurement and analysis [11].

We propose a new framework of combining this representation with 2DPCA based statistical shape models technique for statistically analyzing or classifying 3D anatomic structures. Our 2DPCA based statistical shape model technique is a general method and designed for any shape representations, *e.g.* landmark-based descriptors, medial axes, *etc.* We demonstrate our techniques using real heart data sets extracted from magnetic resonance (MR) images.

## 2. METHODS

Our statistical shape analysis method can be applied into any shape representation approaches (*e.g.* image-based, voxel-based, and surface-based). We are more interested in surface-based approaches which can be applied in more general situations where a surface is not embedded in an image but defined in another way such as segmented boundaries or triangulations. For a 3D volumetric object, its boundary or surface actually defines the shape, and so surface-based representation may be more appropriate for shape study unless the appearance or tissue inside the object is also the focus of interest.

A 3D binary image is reconstructed from each set of 2D segmentation images (*e.g.* MRI, CT, *etc.*), with isotropic voxel values corresponding to whether each voxel is excluded or included. The surface of this 3D binary image is composed of a mesh of square faces. The rest of this section is to introduce the surface description approach using SPHARM expansion, feature reduction method (2DPCA), and point distribution model

### 2.1. SPHARM Shape Description

The SPHARM technique [8] can be used to model arbitrarily shaped, simply connected 3D objects. An input object surface is assumed to be defined by a square surface parameter mesh converted from an isotropic voxel representation. Two steps are involved in converting the object surface to its SPHARM shape description: (1) surface parameterization, and (2) SPHARM expansion.

**Surface parameterization** aims to create a continuous and uniform mapping from the object surface to the surface of a unit sphere. The parameterization is formulated as a constrained optimization problem with the goals of preserving area and topology while minimizing distortions; see [8] for details. The result is a mapping of two spherical coordinates  $\theta$  and  $\phi$  to each point  $\mathbf{v}(\theta, \phi)$  on a surface:

$$\mathbf{v}(\theta, \phi) = \begin{pmatrix} x(\theta, \phi) \\ y(\theta, \phi) \\ z(\theta, \phi) \end{pmatrix}. \quad (1)$$

When the free variables  $\theta$  and  $\phi$  range over the whole sphere,  $\mathbf{v}(\theta, \phi)$  ranges over the whole object surface. **SPHARM expansion** is then used to expand the object surface into a complete set of SPHARM basis functions  $Y_l^m$ , where  $Y_l^m$  denotes the spherical harmonic of degree  $l$  and order  $m$  (see [8] for details). The spherical harmonic basis functions are defined as:

$$Y_l^m(\theta, \phi) \equiv \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\phi},$$

where  $P_l^m(\cos\theta)$  are associated Legendre polynomials (with argument  $\cos\theta$ ) that is defined by the differential equation

$$P_l^m(x) = \frac{(-1)^m}{2^l l!} (1-x^2)^{m/2} \frac{d^{m+l}}{dx^{m+l}} (x^2-1)^l,$$

where  $l$  and  $m$  are integers with  $-l \leq m \leq l$ .

The expansion takes the following form:

$$\mathbf{v}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \mathbf{c}_l^m Y_l^m(\theta, \phi), \quad (2)$$

where

$$\mathbf{c}_l^m = \begin{pmatrix} c_{xl}^m \\ c_{yl}^m \\ c_{zl}^m \end{pmatrix}. \quad (3)$$

The coefficients  $\mathbf{c}_l^m$  are 3D vectors. Their components,  $c_{xl}^m$ ,  $c_{yl}^m$ , and  $c_{zl}^m$  are usually complex numbers. The coefficients up to a user-desired degree can be estimated by solving a set of linear equations in a least square fashion. The object surface can be reconstructed using these coefficients, and using more coefficients leads to a more detailed reconstruction. Thus, a set of coefficients actually form an object surface description.

It is not easy to intuitively understand a SPHARM coefficient, since the coefficient is usually a complex number and provides a measure of the spatial frequency constituents that compose the object. However, the points of the sampled surface (called landmarks) can be considered as a dual representation of the same object. This is a more intuitive descriptor, and so we choose to use this representation in our study.

Using a nearly uniform icosahedron subdivision of spherical surfaces, we obtain a dual landmark representation from the coefficients via the linear mapping described in Eq. (2). Thus, each shape is represented by a set of  $n$  landmarks (*i.e.*, sampling points), which are consistent from one shape to the next.

In all previous statistical shape analysis methods, the shape descriptor becomes a  $3n$  element vector:

$$\mathbf{x} = (x_1, \dots, x_n, y_1, \dots, y_n, z_1, \dots, z_n)^T. \quad (4)$$

Clearly, we have many more dimensions than training objects. PCA was applied to reduce dimensionality to make classification feasible. The previous use Eq. (4) to put all coordinates

into one vector, because the standard PCA only can be performed on a set of vectors. This way really wastes the correlations within all data points and their spatial information. Given a group of  $N$  shapes, the mean shape  $\bar{x}$  can be calculated using

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i,$$

where  $x_i$  is the landmark shape descriptor of the  $i$ -th shape. The covariance matrix  $\mathbf{C}$  of the data as follows:

$$\mathbf{C} = \frac{1}{N-1} \sum_{i=1}^N (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^T, \quad (5)$$

$$\mathbf{C}\mathbf{U} = \Lambda\mathbf{U} \quad (6)$$

where the columns of  $\mathbf{U}$  hold eigenvectors, and the diagonal matrix  $\Lambda$  holds eigenvalues of  $\mathbf{C}$ . The eigenvectors in  $\mathbf{U}$  can be ordered according to respective eigenvalues, which are proportional to the variance explained by each eigenvector. The first few eigenvectors (with greatest eigenvalues) often explain most of variance in the data. Now any shape  $\mathbf{x}$  in the data can be expressed using

$$\mathbf{x} - \bar{\mathbf{x}} = \mathbf{U}\mathbf{V}$$

where  $\mathbf{V}$  is a vector containing the components of  $\mathbf{x}$  in principle components  $\mathbf{U}$ .

Given a dataset of  $m$  objects, the first  $m-1$  principal components are enough to capture all the data variance. Thus,  $\mathbf{V}$  becomes an  $m-1$  element vector, which can be thought of as a new and more compact representation of the shape  $\mathbf{x}$  in the new basis of the deformation modes (*i.e.*,  $\mathbf{x} - \bar{\mathbf{x}}$  is the deformation between an individual shape  $\mathbf{x}$  and the mean  $\bar{\mathbf{x}}$ ). This model is the point distribution model. As we discussed, the dimension of those element vectors are huge and PCA calculation is computational expensive. In order to improve the statistical shape analysis model, we use 2DPCA to reduce features without putting all coordinates of every point into one single vector.

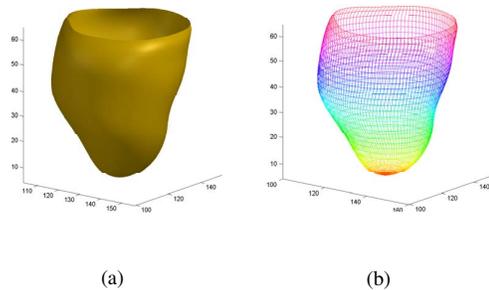
## 2.2. Point Distribution Model using 2DPCA

In 2D approach [4], the coordinates of points do not need to be previously transformed into a vector and are keeping using the natural matrix way, thus a set of  $N$  shapes is represented as  $\{X_1, X_2, \dots, X_N\}$ :

$$X_i = \begin{bmatrix} x_{i1} & x_{i2} & \dots & x_{in} \\ y_{i1} & y_{i2} & \dots & y_{in} \\ z_{i1} & z_{i2} & \dots & z_{in} \end{bmatrix}.$$

2DPCA uses all shapes to construct the shape covariance matrix  $G$  as:

$$G = \sum_{i=1}^N (X_i - \bar{X})^T (X_i - \bar{X}), \quad (7)$$



**Fig. 1.** (a) shows an example of left ventricle; (b) shows the parameterized surface for point distribution model generation.

where  $X_i$  is the  $i$ -th shape with size of  $3 \times n$ ,  $\bar{X}$  is the mean shape of all sample shapes. From Eq. (7), it is obvious to see that  $G$  is an  $n \times n$  non-negative definite matrix. Then, the projection axes of 2DPCA,  $\mathbf{u}_1, \dots, \mathbf{u}_k$  can be obtained by maximizing the matrix scatter criterion:

$$J(\mathbf{u}) = \text{Tr}(\mathbf{u}^T G \mathbf{u}),$$

where  $\mathbf{u}$  is a unitary column vector,  $\mathbf{u}^T \mathbf{u} = 1$ .

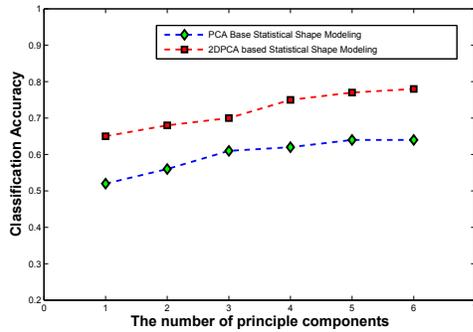
The solution of  $\mathbf{u}_1, \dots, \mathbf{u}_k$  can be obtained by directly solving the algebraic eigenvalue problem  $G\mathbf{u}_i = \lambda_i \mathbf{u}_i$ , where  $\mathbf{u}_i$  is the eigenvector corresponding to the  $i$ -th largest eigenvalue of  $G$ . The 2DPCA solution can be written as:

$$X = YW^T, \quad W = [\mathbf{u}_1, \dots, \mathbf{u}_k],$$

where  $Y$  is the feature matrix of every shape matrix  $X$  and its size is  $3 \times k$ . The original shape matrices  $\{X_1, X_2, \dots, X_N\}$  are projected on to subspace  $U$  and the results are  $\{Y_1, Y_2, \dots, Y_N\}$ . They are the new point distribution model of the  $N$  shapes. The covariance matrix  $G$  of 2DPCA keep the column by column correlations of original matrix which are the exact spatial correlations between different points. The new method can reduce the dimensionality of PDM in a faster way compared to the old PCA based method.

## 3. EXPERIMENTAL RESULTS

In our study, we demonstrate our novel method using the left ventricle shapes. Shape classification results are compared to the old method using standard PCA. The cardiac MRI is used to capture 3D images of a heart during its normal operation in the short-axis or long-axis orientation. With acquisition timed according to heartbeat frequency, a fixed number of images can be acquired during each heartbeat. In this work imaging was performed on a 1.5 Tesla scanner (GE Medical systems) with flip angle  $20^\circ$  and slice thickness of 5 mm. The heart orientation was operator-determined from four-chamber scout views, optimizing for perpendicularity to the cardiac wall. The sequences of heart images were produced in the DICOM format with  $256 \times 256$  pixels size.



**Fig. 2.** The shape classification comparison between PCA based statistical shape models and our new approach.

All images of diastolic stages are segmented for 3D shape reconstruction. When we perform SPHARM for shape reconstruction, the top part of shapes are cut down through fixing the specific angles  $(\theta, \phi)$ . The results are shown in Fig. 1. Two groups data are use: normal group includes 35 subjects and abnormal (with ischemia disease) group includes 28 subjects. All shapes of diastolic hearts are reconstructed using SPHARM method and the PDMs are generated using the old method and our approach. To evaluate the performance, K-NN classifiers with  $K=1$  are employed to both PDMs. In our method, the distance between two PDMs ( $A = (a_{ij})_{3 \times k}$  and  $B = (b_{ij})_{3 \times k}$ ) are defined as follows:

$$d_F(A, B) = \|A - B\|_F = \left( \sum_{i=1}^3 \sum_{j=1}^k (a_{ij} - b_{ij})^2 \right)^{1/2}.$$

Fig. 2 plots the shape classification results of both the traditional statistical shape models and our approach. The values on  $x$ -axis are the number of principle components used in shape classification and the values on  $y$ -axis are the classification accuracy. Our method always has a better performance than the previous one, because more spatial relations between points are considered when the program selects the principle components (we select more meaningful principle components).

#### 4. CONCLUSION

In this paper, we propose a novel statistical shape models method with naturally representing the shape parameters into matrix, not forcing them into a vector as previous approaches. Using such matrices, the new statistical shape models obtain the spatial correlations between points faster than the previous PCA based method. 2DPCA are used to map PDMs onto subspaces with lower dimensionality. Our method selects more meaningful principle components compared to others. The experimental results of shape classification using real cardiac

left ventricle shapes validate our novel method outperforms the previous PCA based statistical shape models.

#### 5. REFERENCES

- [1] T.F. Cootes, C.J. Taylor, D. Cooper, and J. Graham. "Active Shape Models - their training and application." *Computer Vision Image Understanding*, 61(1): 38-59, 1995.
- [2] Cootes, T. F., A. Hill, C. J. Taylor, and J. Haslam. "The use of Active Shape Models for locating structures in medical images." *Image and Vision Computing*, 12: 355-366, 1994.
- [3] Kotcheff, A. C. W. and C. J. Taylor. "Automatic Construction of Eigenshape Models by Direct Optimisation." *Medical Image Analysis*, 2: 303-314, 1998.
- [4] J. Yang and D. Zhang and A. F. Frangi and J. Yang. "Twodimensional pca: A new approach to appearancebased face representation and recognition." *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(1), 2004.
- [5] Martin Styner, Guido Gerig. "Three-Dimensional Medial Shape Representation Incorporating Object Variability." In *Proc. IEEE Conf. Computer Vision and Pattern Recognition*, pp. 651-656, 2002.
- [6] A. Matheny, D.B. Goldgof. "The use of three- and four-dimensional surface harmonics for rigid and nonrigid shape recovery and representation." *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 17(10):967-981, 1995.
- [7] Chang Wen Chen, T.S. Huang, M. Arrott. "Modeling, analysis, and visualization of left ventricle shape and motion by hierarchical decomposition." *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 16(4):342-356, 1994.
- [8] Ch. Brechbühler, G. Gerig, and O. Kübler. "Parametrization of closed surfaces for 3D shape description." *Computer Vision and Image Understanding*, 61(2):154-170, 1995.
- [9] G. Gerig, and M. Styner. "Shape versus Size: Improved Understanding of the Morphology of Brain Structures," *4th International Conference on Medical Image Computing and Computer Assisted Intervention*, LNCS 2208:24-32, 2001.
- [10] A. Kelemen, G. Szekely, and G. Gerig. "Elastic Model-based Segmentation of 3-D Neuroradiological Data Sets." *IEEE Trans. on Medical Imaging*, 18(10):828-839, 1999.
- [11] H. Huang, L. Shen, F. Makedon, B. Hettleman and J. Pearlman. "Surface Alignment of 3D Spherical Harmonic Models: Application to Cardiac MRI Analysis." *International Conf. on Medical Image Computing and Computer Assisted Intervention*, pp. 67-74, 2005.